LABORATORY STUDIES IN ASSET TRADING:

PART IV--Monte Carlo Experimentation with the Pure Double-Auction Market Model*

by

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In this report, we explore the implications of Model VI of the preceding report. Model VI can be depicted a double-auction model wherein potential buyers compete with other buyers, sellers compete with sellers, and transactions occur only between those offering the best prices to the other side. The dynamics of the model were described by the set of partial differential equations (24) of the preceding report (Part III). Since these equations appear to be quite difficult to analyze directly, we turn to Monte Carlo methods in order to gain certain qualitative and quantitative insights into the model. Of particular interest are characteristics of the transaction price and price change distributions implied by the model.

As in most models of its complexity, several parameters must be set in order to completely specify Model VI. In particular, it is necessary to set the ratio of order arrival rates to the order extinction rate, and the selection function (see formula (22), Part III). Unfortunately, we can give no guarantees that the parameter values used in the Monte Carlo studies reported below are in any way representative of values that might be determined upon analysis of actual market data (where such exists); the writer has simply tried to make these values "reasonable" with regard to the casual data available to him. Hence the insights drawn below must be regarded as only hypotheses, not conclusions; and the most important of these will be qualitative in nature.

MONTE CARLO RUN 1

Parameters

Values: $\lambda = \mu = 5$; $\nu = 1$; $K = 7$; $p_i = i$ and $b_i = S_i = 1/7$, $i = 1,2,$

..., 7 (uniformly distributed selection function); sample size = 5000 .

As in all runs, the prices p_i were set equal to the integers $i = 1,2,\ldots,K$. This approach does not imply much loss of generality, since (a) most of our results are independent of the actual scale and location of possible prices, and (b) trading in the organized securities exchanges is in fact done at prices which are equally spaced, namely, multiples of 1/8 of a dollar. Also, we assume that buy and sell orders arrive at the same rate $(\lambda = \mu)$ in all runs.

In this first run, order arrivals take place at a rate 10 times faster than the rate of order extinction $\{(\lambda+\mu)/\nu = (5+5)/1\}$. The "book," i.e., the set of unexecuted active orders, will tend to be quite sparse since in equilibrium orders would extinguished at approximately the same rate they arrive, assuming no transactions were to occur. Therefore the expected number of orders on the book is bounded above by 10, because transactions will in fact occur to delete some orders besides those that expire.

Results

The transaction price change frequency distribution for 4978 transactions (some early transactions were ignored to avoid including gross transient behavior) was as follows:

Δ p	# occurrences
-6	11
-5	55
-4	159
-3	290
-2	603
-1	907
0	932
+1	927
+2	555
+3	312
+4	156
+5	61
+6	10
	4,978

TABLE 1

There seems to be nothing unusual about this distribution, other than the fact that it is not as peaked as available market data suggest it should be (cf., Niederhoffer and Osborn [, p.]); presumably this is partially due to the widely spread uniform selection function.

Of more interest is the intertransaction time distribution whose observed shape is described in Figure 1.

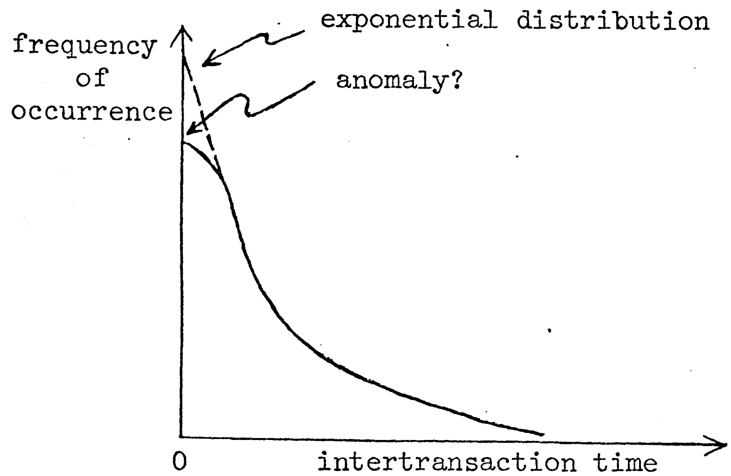

Figure 1

The transaction event process is embedded in the order arrival event process, the latter of which is specified to be Markovian in nature. Although the intertransaction time (time between successive transactions) distribution appears to be approximately exponential in its right-hand tail, it fails to give quite enough weight to the small intertransaction times (relative to the exponential distribution) as illustrated by the anomaly near the origin in Figure 1. We conjecture then that the intertransaction times are <u>not</u> Markovian, and offer the following explanation: When a transaction occurs, it can only widen the spread (the numeric difference between ask and bid prices) or leave it unchanged. When the spread is wider, it takes longer on the average for the next transaction to occur; moreover, the mere passage of time makes a transaction in the next instant more probable, because in the meanwhile untransacted orders may have accumulated to narrow the spread. If this explanation is accurate, we would then expect to see the non-Markovian anomaly near the origin appear more pronounced in situations where the book is sparse (e.g., for low $(\lambda + \mu)/\nu$ ratios). Conversely, intertransaction times should appear to be more nearly Markovian in nature when the book consistantly remains full.

MONTE CARLO RUN 2

Parameters

Values: $\lambda = \mu = 50$; $\nu = 1$, $K = 7$, $p_i = i$ and $b_i = s_i = 1/7$, $i = 1,2,\ldots,7$ (uniform); sample size = 5000 transactions.

This run differs from the first only insofar as the order arrival rates (λ, μ) are concerned. With the increase of the ratio $\frac{\lambda+\mu}{\nu}$, we would expect the book to be more densely packed with orders, so that the transaction price change distribution should be more peaked than that of Table 1.

Results

As Table 2 shows, the price change distribution is in fact more peaked:

Δp	# occurrences
-6	1
-5	11
-4	86
-3	239
-2	526
-1	976
0	1,276
+1	1,023
+2	506
+3	236
+4	82
+5	16
+6	0

TABLE 2

Also, as we conjectured, the non-Markovian anomaly in the intertransaction time distribution appeared to be much less pronounced.

MONTE CARLO RUN 3

Parameters

Values: $\lambda = \mu = 50$; $\nu = 1$; $K = 8$; $p_i = i$, $i = 1,2,\ldots,8$;
$(b_i,\ i = 1,2,\ldots,8) = (3/57,\ 5/57,\ 7/57,\ 9/57,\ 17/57,\ 12/57,\ 3/57,\ 1/57)$;
$(s_i,\ i = 1,2,\ldots,8) = (1/57,\ 3/57,\ 12/57,\ 17/57,\ 9/57,\ 7/57,\ 5/57,\ 3/57)$;
sample size = 10,000 transactions.

In this run, more realistic selection functions were contrived. The general idea was to make these represent closely price-competitive buyers and sellers, as illustrated in Figure 2:

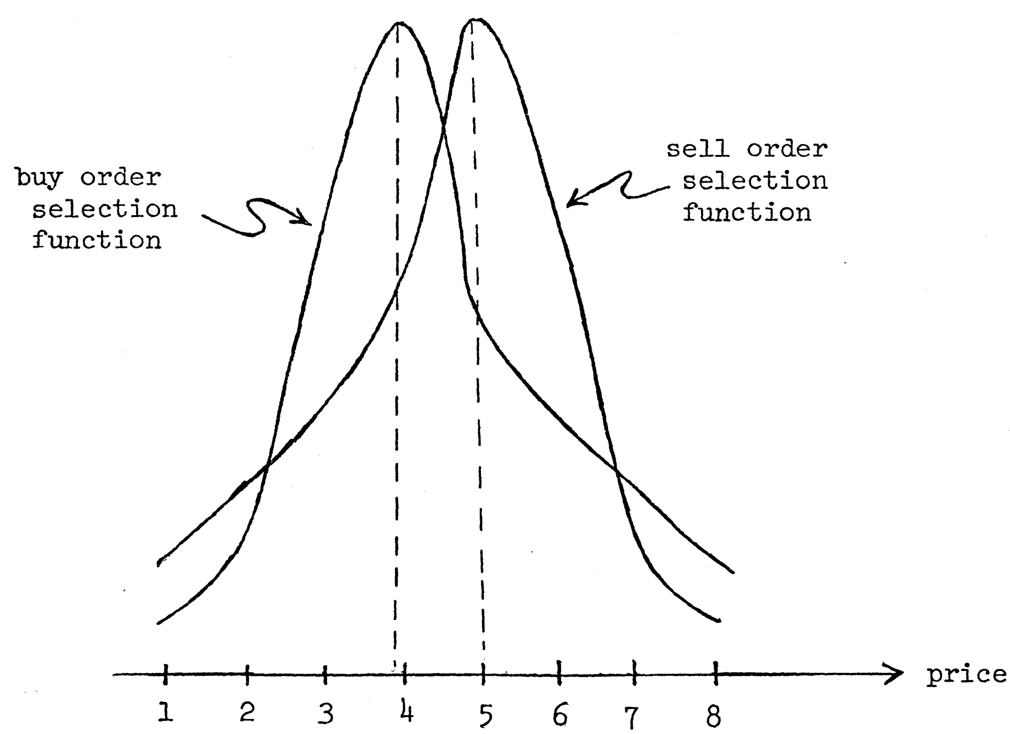

Figure 2

An effect which appeared in every one of the Monte Carlo runs is well illustrated by this particular run, namely, certain serial dependence phenomena in the transaction price change series. In order to assess the data generated from this model in the appropriate light, we first reproduce the empirical one-stage serial dependence table observed by Niederhoffer and Osborne in a sample of 10,536 price change observations on the first seven stocks in the Dow Jones Industrial Averages:

Δp_{t-1} (in 1/8's)	Δp_t (in 1/8's)							totals
	-3	-2	-1	0	+1	+2	+3	
-3	0	0	3	9	3	4	2	21
-2	1	10	32	136	61	51	1	292
-1	0	35	231	1,059	777	80	3	2,185
0	4	130	1,128	3,139	1,041	130	3	5,575
+1	9	72	709	1,104	236	22	4	2,156
+2	5	48	64	129	40	6	1	293
+3	2	2	2	6	1	1	0	14
totals	21	297	2,169	5,582	2,159	294	16	10,536

Table 3: Empirical frequency table of consecutive pairs of price changes (from Niederhoffer and Osborne).

Table 3 shows the number of occurrences where a price change of Δp_{t-1} was followed by a succeeding price change of Δp_t .

The same type of table was constructed for Model VI with the previously mentioned parameter settings. The results are given in Table 4:

Δp_{t-1}				Δp_t				totals
	-3	-2	-1	0	+1	+2	+3	
-3	0	0	1	16	31	13	22	83
-2	0	0	38	235	197	158	21	649
-1	0	16	296	889	907	294	26	2,428
0	14	157	952	1,384	960	160	17	3,644
+1	25	290	897	908	288	20	1	2,429
+2	26	167	214	190	48	0	0	645
+3	19	19	29	20	0	0	0	87
totals	84	649	2,427	3,642	2,431	645	87	9,965

Table 4: Monte Carlo frequency table of consecutive pairs of price changes, Model VI, run 3. (Eleven observations of $\Delta p = \pm 4$ were deleted for ease of comparison with Table 3.)

Qualitatively speaking, Tables 3 and 4 have some remarkable similarities. A strong degree of negative serial correlation is present in both, i.e., there is a tendency toward "reversals" ($\Delta p_{t-1} < 0$ followed by $\Delta p_t > 0$, or $\Delta p_{t-1} > 0$ followed by $\Delta p_t < 0$) rather than "continuations" ($\Delta p_{t-1} > 0$ then $\Delta p_t > 0$, or $\Delta p_{t-1} < 0$ then $\Delta p_t < 0$). Where Niederhoffer and Osborne specifically note that a reversal involving 1/8 point changes is three times as likely as a continuation of 1/8 point changes in Table 3, the same can be said of Table 4. Indeed, the Monte Carlo results would seem to strongly support Niederhoffer and Osborne's contention that the specialist's book of orders is primarily responsible for the observed pattern of reversals on the New York Stock Exchange. (Conversely, Model VI is thus apparently the only available quantitative model which successfully predicts this observed pattern of reversals, at least to the present writer's knowledge. But we must be wary here, because although the model is apparently sufficient to explain the first-order market dependencies, it is certainly not necessary.)

QUESTIONS FOR FURTHER STUDY

Although the computing resources applied in the reported three Monte Carlo runs were considerable, it is discouraging to note that their results seem to raise more questions than they answer. Among these are:

1. Simple n-th order correlations or frequency tables appear to be simply adequate to capture the subtle, complex nature of serial dependency in the double-auction model. How much more inadequate are they vis-a-vis real market data?

2. How persistent is the serial dependence introduced by the auction market? Could it last over weeks or months in, say, the NYSE? Does the subtle nature of this dependency have something to do with the observed leptokurtosis of market price-change distributions? What effect does it have on the Central Limit Theorem arguments that are often advanced to justify the normal or stable-Paretian distributions as the limiting distributions of price changes?

3. Can it be that intertransaction time distributions are truely non-Markovian in real-world markets?

4. What are the parameter values which best make Model VI approximate observed data? Are there other forms of data that are much more relevant to identifying the parameters of auction markets than those forms most commonly recorded? What new measurement process are needed?