LABORATORY STUDIES IN ASSET TRADING:

PART V--Market Performance Measures*

by

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INTRODUCTION

In this report we shall define various measures of market behavior which are termed "operating characteristics" in a preceding report of this series. The purpose of such definitions is to arrive at quantities which reflect on the extent to which particular markets serve social welfare goals. Our method shall be to review the existing measures in each case, and then offer critiques and refinements of these. Also, new measures will be introduced for discussion.

All reasonable measures of market performance, i.e., operating characteristics, must satisfy at least two criteria: relevance and operationality. The relevance of an operating characteristic has to do with the way in which it will be used, which is a matter of intent on the part of the user. A primary question here is then "does the operating characteristic in fact measure that which is intended to be measured?" Operationality has to do with the description of the measurement process implied, particular with respect to absence of ambiguity in that description. "Are the quantities observable?", "Can they be unambiguously aggregated?" and similar questions are central to the operationality criterion.

In what follows, I shall suggest certain market measures. The terminology used in each is based upon the concept that a market has (1) a "current state description" at any time t and (2) a "history" that consists of the time-ordered sequence of state changes, or "events," which are assumed to take place at discrete epochs in time. In particular, we assume that a state description includes at any time t a set $B(t)$ consisting of "active buy orders" and a set $S(t)$ of "active sell orders." $B(t)$ and $S(t)$ are each assumed to be composed of triples of the form (p, q, c) where p is a price, q is a quantity, and c is a vector of all other descriptive attributes which condition the order. Let $R(t)$ be a set of structural variables and their values at time t which govern such things as the precedence of orders, the market history (e.g., sign of the "tick"), and other joint variables of interest. Then the state description of a market is the triple $M(t) = \{B(t), S(t), R(t)\}$. As additional terminology, let

$$b(t) = \begin{cases} \max\{p_i \mid (p_i, q_i, c_i) \in B(t)\} & \text{if } B(t) \neq \emptyset \\ -\infty & \text{if } B(t) = \emptyset \end{cases} \tag{1}$$

$$a(t) = \begin{cases} \min\{p_i \mid (p_i, q_i, c_i) \in S(t)\} & \text{if } S(t) \neq \emptyset \\ +\infty & \text{if } S(t) = \emptyset \end{cases} \tag{2}$$

$b(t)$ and $a(t)$ are the "bid and ask" prices, respectively.

Events, or state description changes, take place primarily because of change in $B(t)$ and $S(t)$, that is, new orders arrive, existing expire or are cancelled, etc. In addition, whenever $b(t) \geq a(t)$, transactions may occur, which we shall assume take place instantaneously upon order arrival events. When a transaction does take place, its impact may be to delete at least one order from $B(t)$ or $S(t)$ or both. We assume now that transaction events occur at epochs $T_1^* < T_2^* < \ldots < T_n^*$ on the time scale and at prices $p_1^*, p_2^*, \ldots, p_n^*$, the quantities $q_1^*, q_2^*, \ldots q_n^*$ have been exchanged. (For reasons discussed in a successor to this report of the present series, we will consider these quantities as if they were random variables.)

CONTINUITY

SEC regulations charge the organized securities exchanges with maintaining market "continuity." This term is commonly interpreted to mean lack of volatility, or a closeness of one transaction price to the next. More formally, define $\Delta p_i \triangleq p^*_{i+1} - p_i$, $i=1,2,\ldots,n-1$ as the "price change," and let

$$F_n(u) \triangleq \frac{1}{n-1} \sum_{i=1}^{n-1} G(\Delta p_i, u) \qquad (3)$$

where

$$G(\Delta p_i, u) = \begin{cases} 1 & \text{if } |\Delta p_i| \le u \\ 0 & \text{if } |\Delta p_i| > u \end{cases} \qquad (4)$$

$F_n(u)$ thus represents the empirical cumulative distribution function (c.d.f.) of absolute price changes. Certain values of this function are used by the securities exchanges as operating characteristics. In particular, since the smallest unit of price is 1/8 of a dollar, $F_n(1/8)$ and $F_n(1/4)$ are commonly measured by the U.S. securities exchanges. (Typical values of these quantities are in the neighborhoods of .7 and .9, respectively.)

A high value of this continuity measure is widely felt to represent a social good. The reasoning goes that all investors are better off because they thereby possess increased certainty about the near-future prices at which they may buy or sell their assets, given knowledge of current prices. Yet it is clear that the value of such knowledge may be far outweighed by what is given up to achieve price continuity, at least for some investors. For example, consider the case where news favorable to holding an asset is received by all investors and thus assume that its absolute price level would increase several points in the absence of continuity-maintaining mechanisms. The imposition of such mechanisms would then favor buyers at the expense of sellers. In such a case, sellers might certainly be willing to give up price certainty (and an abnormally low selling price) in exchange for a higher (but less certain) selling price. At any rate, it would take a reasonably perverse social welfare function to view such a resource transfer from sellers to buyers as socially more desirable.

One criticism of the continuity measure (3) is that in some real-world exchanges, certain exchange members are in fact legally charged with maintaining "continuous" markets, but, it is argued, they may circumvent the intent of the measure. This is alleged to be done by adjusting their trading quantities in the course of several trades, each of which may involve a minimal price change. To illustrate the nature of this possibility, let us assume that the unconstrained level of some asset, given some news, would instantaneously jump by an amount $5\delta (\delta > 0)$, but that strict continuity controls the form $|\Delta p_i| \le \delta$ are inposed. Now the "exchange member," being also charged with the obligation of making a market (being willing to either buy or sell at any instant), might have an incentive to meet this obligation as a buyer or as a seller via differing quantity strategies, as illustrated by the example below:

Price	Members' Quantity Strategy (in "shares")	
	As buyer	As seller
p_i^*		
$p_{i+1}^* = p_i^* + \delta$	500	100
$p_{i+2}^* = p_i^* + 2\delta$	400	100
$p_{i+3}^* = p_i^* + 3\delta$	100	100
$p_{i+4}^* = p_i^* + 4\delta$	100	200
$p_{i+5}^* = p_i^* + 5\delta$	100	700
	1200	1200

In each case, the exchange member has provided or absorbed 1200 "shares" of the asset, while maintaining perfect price continuity. However, the resulting price level "certainty" is in fact illusory to nonmembers: what is certain to any one nonmember is that the next transaction price will at most δ removed from the previous transaction price, <u>but not that he himself will be able to participate in the transaction that takes place at that next price</u>. The "nonmembers" in this example will have an increased chance of participation on the "wrong" side of a transaction (buying high, selling low) and a decreased change of participation on the right side of a transaction.

In other words, alternative continuity measures might reasonably take trading volume into account. As an example, consider the measure

$$C_n(u) = \frac{1}{n-1} \sum_{i=1}^{n-1} H(\Delta p_i, q^*_{i+1}, u) \tag{5}$$

where

$$H(\Delta p_i, q^*_{i+1}, u) = \begin{cases} 1 & \text{if } \dfrac{|\Delta p_i|}{q^*_{i+1}} \leq u \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

Formula (5) thus gives the empirical c.d.f. of the absolute price changes inversely weighted by quantity traded. It may be viewed as partially overcoming the objection noted above.

However, other criticisms of (3) can be leveled at the widely used continuity measure. For one thing, it deals in absolute price changes rather than in percentage changes. Since percentage change is more closely related to investors' perceptions of risk and return from an asset, we can expect some anomalous behavior in the measure. For example, those exchanges which trade in low-priced assets (e.g., the AMEX) could be expected to maintain better "continuity" measures than those which deal in high-priced ones (e.g., the NYSE) solely for this reason. Trading should be more "continuous" after a stock split than before, and so on.

Another criticism of (3) is that it takes no account of the calendar time between succeeding transactions. Intuitively, we feel that an asset price is more "volatile" if, say, successive 1/4 point transaction price changes occur at the rate of one per minute than if the same changes take place at the rate of one per hour. A measure that takes this, as well as percentage change, into account is

$$D_n(u) = \frac{1}{n-1} \sum_{i=1}^{n-1} L(\Delta p_i, q_{i+1}^*, p_i^*, \Delta t_i, u) \tag{7}$$

where

$$L(\Delta p_i, q_{i+1}^*, p_i^*, \Delta t_i, u) = \begin{cases} 1 & \text{if } \dfrac{|\Delta p_i| e^{-\Delta t_i}}{p^* q_{i+1}^*} \leq u \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

and $\Delta t_i = T_{i+1}^* - T_i^*$. Alternatively, we might use a more direct and dimensionless measure like

$$E_n(u) = \frac{1}{(n-1)\left(\sum_{i=1}^{n-1} \frac{1}{\Delta t_i q_{i+1}^*}\right)} \sum_{i=1}^{n-1} \frac{|\Delta p_i|}{\Delta t_i q_{i+1}^* p_i^*} \tag{9}$$

The basic point here is that the traditional continuity measure (3) is not the only conceivable one, and indeed may be dominated on a priori grounds by other measures. However, the ultimate test of a quantitative definition includes not only prior considerations but also empirical consistency and usefulness vis-a-vis other definitions and relationships that comprise a theory. Since we are not yet prepared to undertake the latter, we shall simply continue here to point out various alternatives to the traditional market operating characteristics.

CLOSENESS

The "closeness" of a market is usually thought of as the spread between the bid and ask prices, i.e., $a(t) - b(t)$. The closer this spread is, the less of a premium potential buyers and sellers would seem to be paying simply for the opportunity of trading. Hence closeness seems to represent a social good to the investors, a "competitiveness" of the market in an economic sense.

The basic problem with the measure seems not to be its intuitive appeal, but rather the technical difficulties of aggregating it over periods of time. The measure as given, $a(t) - b(t)$, represents a snapshot taken at the instant t in time. In order to make meaningful statements in comparing "closeness" in two alternate market structures, we must insist on some form of temporal aggregation. The most obvious approach is to adopt the quantity

$$V = \frac{1}{T} \int_{0}^{T} [a(t) - b(t)] \, dt \qquad (10)$$

as an aggregate over any time interval $[0,T]$ wherein $a(t) - b(t) \neq \infty$. The writer is aware of no attempts on the part of any real-world exchange to directly measure this quantity; however, it seems to be approximated (with uncertain accuracy) by schemes that sample $a(t) - b(t)$ periodically, say, at the close of trading each day, and averaging. (Indeed, it is not now known whether such averages are valid, or whether (10) even converges to a stable limit as $T \to \infty$. The infinite-moment theories of stock price movements might admit the contrary result!)

A special difficulty arises when $a(t) - b(t) = \infty$, i.e., when no bid or no offer is present. Since we cannot give these occurrences any weight in (10), it seems reasonable to leave (10) as given and instead record the percentage of time this state exists. That is, for any interval of time $[0,T]$, let

$$W = \frac{1}{T} \int_0^T \left\{ \begin{array}{ll} 1 & \text{if } a(t) - b(t) = \infty \\ 0 & \text{otherwise} \end{array} \right\} dt \qquad (11)$$

be defined, which gives the desired measure.

STABILIZATION

Stabilization is a concept which depends inherently upon discrimination between two classes of market participants, which for our purposes we may again think of as exchange "members" versus "nonmembers." Members are believed to exert a stabilizing influence on asset prices if they purchase in a trade at a price lower than the preceding transaction price or sell in a trade higher than the preceding transaction price. Such activity by members is widely believed to be beneficial to nonmembers. Indeed, the major securities exchange of the world, the NYSE, depends that certain of its members maintain at least a 75% "tick test." This measure is quite simple, being just that fraction of a member's total trades which were stabilizing over some period of time.

The measure may be criticized on the grounds that it again allows quantity strategies to defeat its intent. That is, it would not be impossible for members to trade in small quantities within stabilizing transactions and in large quantities within de-stabilizing transactions. A simple remedy here is to weight the stabilizing and de-stabilizing trade by quantity traded and then determine the analogous fraction.

On the other hand, there are those who argue that stabilization is entirely consistent with profit maximizing behavior on the part of exchange members. If this is true, then there would seem to be no incentive for defeating the tick test. But there would also be little by way of justification member stabilization as a benefit to nonmembers. Later research in this series will hopefully assist in resolving this issue.

DEPTH

"Depth" is defined by the AMEX as the quantity of an asset traded at a single price. That is, we consider subseries of the transaction price series $p_1^*, p_2^*, \ldots, p_n^*$. p_i^* and p_{i+1}^* are classified as being within the same subseries if and only if $p_i^* = p_{i+1}^*$. Suppose there are k such subseries whose index sets are $I_1, I_2, \ldots, I_k$. Then the "depth" operating characteristic is given by

$$X = \frac{1}{k} \sum_{j=1}^{k} \sum_{i \in I_j} q_i^* . \tag{12}$$

High values of X are apparently felt to be a social good by some observers, but we shall pass no judgments here other than to remark that the measure does not capture all the qualities evoked in the present writer's understanding of the term "depth." To capture these, we instead introduce the following operating characteristics.

"LIQUIDITY"

With these measures, we shall attempt to capture the extent to which the forced sale or purchase of certain amounts of an asset would be absorbed by the market at various prices. Let $\bar{p}(t) = [a(t) + b(t)]/2$ be defined as a matter of convenience. We might then term

$$Z_B^{(\alpha)} = \sum_{(p_i,q_i,c_i)\epsilon B(t)} q_i e^{-\alpha\{\frac{|\bar{p}(t)-p_i|}{p(t)}\}} \tag{13}$$

and

$$Z_S^{(\alpha)} = \sum_{(p_i,q_i,c_i)\epsilon S(t)} q_i e^{-\alpha\{\frac{|\bar{p}(t)-p_i|}{p(t)}\}} \tag{14}$$

as the "buying liquidity" and "selling liquidity," respectively. Basically, these quantities total up the quantities currently bid for or offered, discounted by their percentage price distance from the average of the current bid and ask prices. (We of course do not claim these to be the only reasonable measures capturing such notions, but see no harm in at least temporarily adopting these for discussion.)

In addition, we could term $Z^{(\alpha)} = Z_B^{(\alpha)} + Z_S^{(\alpha)}$ "total liquidity." It would seem that a case could be made for ascribing social worth to high values of this variable. Also, let $Y^{(\alpha)} = Z_B^{(\alpha)} - Z_S^{(\alpha)}$; then we can add to our earlier list of null hypotheses the following: exchange members will be buyers or sellers independent of the sign and magnitude of $Y^{(\alpha)}$.

EFFICIENCY

"Efficiency" is currently one of the more fashionable concepts to be applied to markets in recent research; it also demands more by way of assumptions. The notion of efficiency is based on the idea that real-world markets are "fair games." More precisely, consider epochs $0, 1, 2, \ldots, m$ on the time line which divide it into m intervals of time. Let p_t be the "price" of a unit of some asset at epoch t and let $\tilde{d}_t$ be the amount of yield ("dividends") from holding that unit between epochs $t-1$ and t . Define

$$\tilde{r}_t \equiv \frac{\tilde{p}_t - p_{t-1} + \tilde{d}_t}{p_{t-1}} \qquad (15)$$

as the return from a unit of the asset between epochs $t-1$ and t , treating all quantities with a tilda as random variables. The fair game hypothesis states that

$$E_{ep}\left[\tilde{r}_t - E_{ea}[\tilde{r}_t | \Omega_t]\right] = 0 \qquad (16)$$

where Ω_t is an information set available at t , E_{ea} is an <u>ex ante</u> expectation, and E_{ep} is an <u>ex post</u> expectation. The hypothesis is usually then classified according to its scope, the nature of the information set, and the type of ex ante expectation employed. (For example, if Ω_t is limited to historical prices, this is called the "weak" form of the hypothesis. Various models, e.g., the "market model," are used in computing ex ante expectations, and so forth.)

Like the other operating characteristics discussed in this report, market efficiency measures will be available in the forthcoming laboratory research. In particular, the laboratory environment will be used to carefully control the information sets of market participants. Also, there are some exciting possibilities for calculating ex ante expectations. One of these is to simply ask all market participants about what their price expectations are for the future period. It may be possible to identify the outstanding laboratory subjects insofar as their price-predictive abilities and thus shed a good deal of light on the "strong" forms of the efficient market hypothesis.